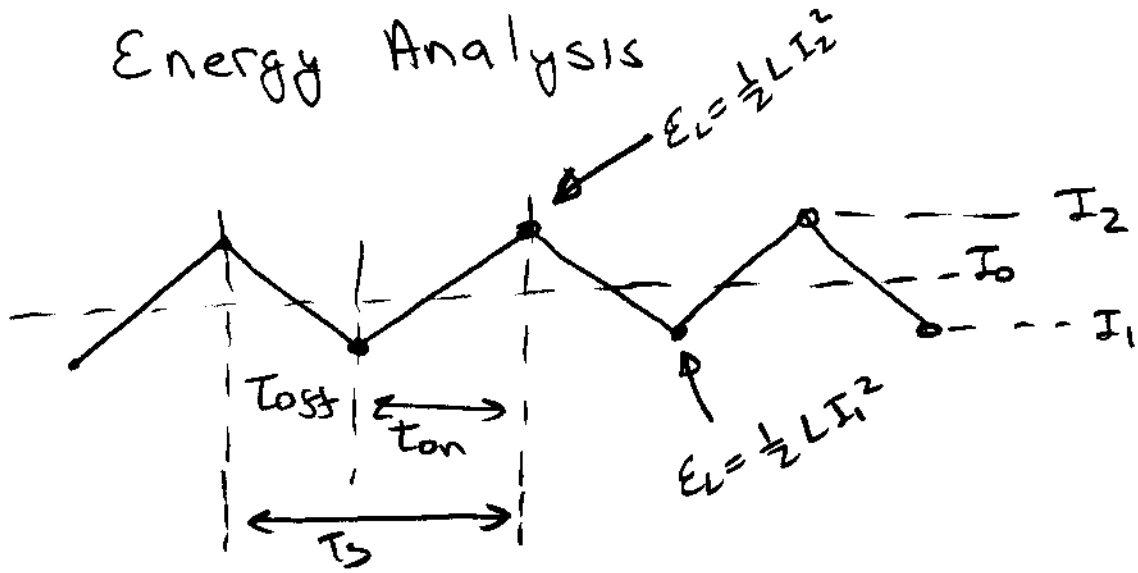
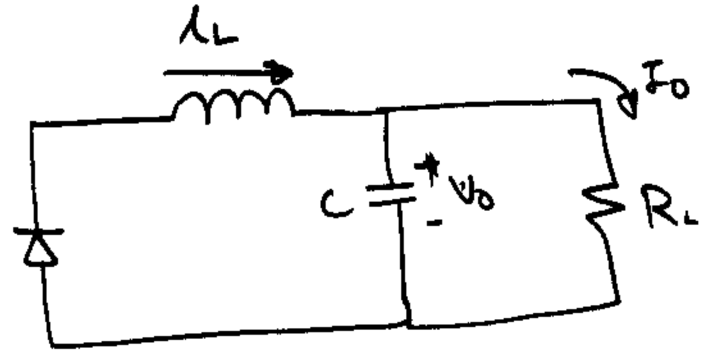
Energy Analysis



During toff



- During toff, $I_2 \rightarrow I_1$

Energy in inductor is delivered to R_L

$$\frac{1}{2} L I_2^2 - \frac{1}{2} L I_1^2 = V_o I_o t_{off}$$

We know that $\frac{I_2 + I_1}{2} = I_o$

$$\frac{1}{2} L (I_2^2 - I_1^2) = \frac{V_o t_{off}}{2} (I_1 + I_2)$$

$$\frac{1}{2} L (I_2 + I_1)(I_2 - I_1) = \frac{V_o t_{off}}{2} (I_1 + I_2)$$

$$\frac{1}{2} L (I_2 - I_1) = \frac{V_o t_{off}}{2} \quad (1)$$

and $\quad t_{off} = T_s - t_{on}$

and $\quad V_o = \frac{t_{on}}{T_s} V_d \quad \Rightarrow T_s = t_{on} \frac{V_d}{V_o}$

$$\Rightarrow t_{off} = t_{on} \frac{V_d}{V_o} - t_{on} = t_{on}\left(\frac{V_d}{V_o} - 1\right)$$

Sub into (1)

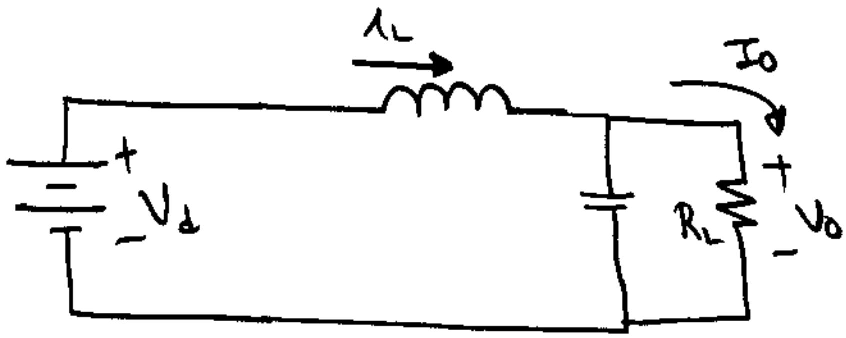
$$L (I_2 - I_1) = V_o t_{on} \left(\frac{V_d}{V_o} - 1\right)$$

OR

$$\boxed{I_2 - I_1 = \frac{V_d - V_o}{L} t_{on}}$$

Same as before

During t_{on}



During t_{on} $i_L \rightarrow I_1$ to I_2

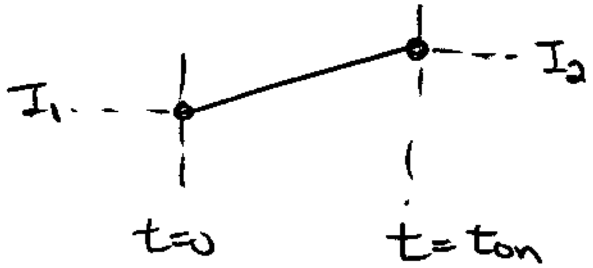
Energy dissipated by R_L = $V_o I_o t_{on}$

Energy stored in inductor: $\frac{1}{2} L I_2^2 - \frac{1}{2} L I_1^2$

Both of these energies must be supplied By V_D

$$\mathcal{E}_{V_d} = \int_0^{t_{on}} P_{V_d}(t)\,dt = \int_0^{t_{on}} V_d\, I_L(t)\,dt$$

$$I_L(t) = I_1 + \left(\frac{I_2 - I_1}{T_{on}}\right) t$$



$$\mathcal{E}_{V_d} = V_d \int_0^{t_{on}} \left(I_1 + \frac{I_2 - I_1}{t_{on}} t\right) dt$$

$$= V_d \left[I_1 t + \frac{(I_2 - I_1)}{t_{on}} \frac{t^2}{2} \right]_0^{t_{on}}$$

$$= V_d \left[I_1 t_{on} + \left(\frac{I_2 - I_1}{2}\right) t_{on} \right] = V_d \left[\frac{I_1 + I_2}{2} t_{on} \right]$$

Next, Energy balance:

Energy Supplied by Source = additional Energy stored in L + Energy dissipated in R_L

$$V_D\left[\frac{I_1+I_2}{2}\right]t_{on} = \frac{1}{2}L\left[I_2^2 - I_1^2\right] + V_o I_o t_{on}$$

Sub in $I_o = \dfrac{I_1+I_2}{2}$

$$V_D\left[\frac{I_1+I_2}{2}\right]t_{on} = \frac{1}{2}L\left[I_2+I_1\right]\left[I_2-I_1\right] + V_o t_{on}\left[\frac{I_1+I_2}{2}\right]$$

OR

$$(V_D - V_o)t_{on} = L(I_2 - I_1)$$

OR

$$\boxed{I_2 - I_1 = \frac{V_D - V_o}{L}\, t_{on}}$$

Same as before

Choosing the Capacitor

Method 1

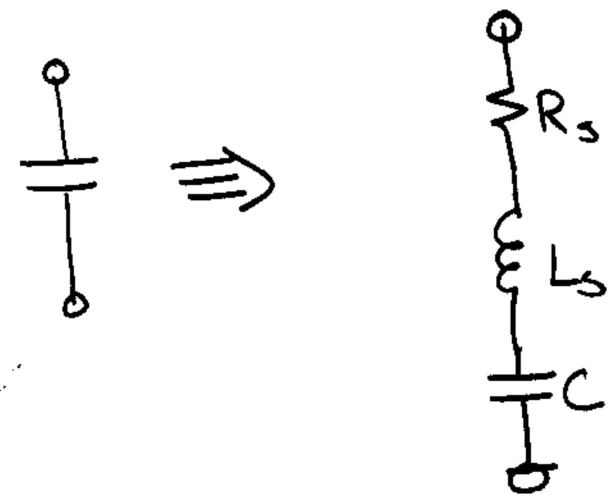
- Design L + C as Low pass Filter

Choose $F_c < \frac{F_s}{100}$

Then $F_c = \frac{1}{2\pi\sqrt{LC}}$

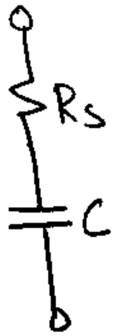
Method 2

Use a Real model of a Capacitor



R= equivalent Series Resistance (ESR)

Below 300K - 500 KHz, Ignore L_S



- Two Ripple components due to C & R_s
- The Two Ripple components are out of phase, but for worst case analysis, we shall Assume they add in phase.

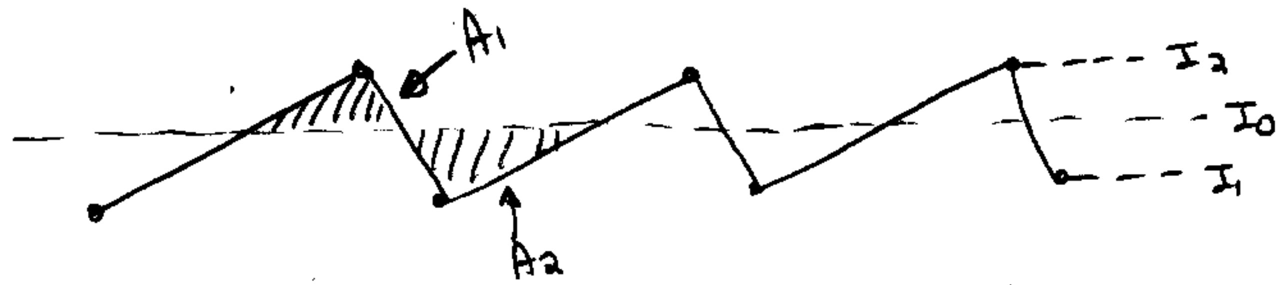
- For Typical Electrolytic R_sC = const

- Typical Values For R_sC are in the Range 50 to 80×10^{-6} μS

Ripple Voltage due to R_s is

$$V_{RR} = (I_2 - I_1) R_s$$

Find the Capacitive Ripple



I_o = Load Current $= \dfrac{I_1 + I_2}{2}$

During A_1:

- current larger than I_o
- extra current goes into cap.
- cap voltage increases

During A_2:

- current smaller than I_o
- cap supplies current to load
- cap voltage decreases

Note: $A_1 = A_2$

During A_1, the average value of current through L is

$$\frac{I_2 + I_0}{2} = \frac{I_2 + \left(\frac{I_1+I_2}{2}\right)}{2} = \frac{I_2}{2} + \frac{I_2}{4} + \frac{I_1}{4}$$

$$\langle I_C \rangle = \langle I_L \rangle - I_0$$

$$\langle I_C \rangle = \frac{I_2}{2} + \frac{I_2}{4} + \frac{I_1}{4} - I_0$$

$$= \frac{I_2}{2} + \frac{I_2}{4} + \frac{I_1}{4} - \left(\frac{I_1+I_2}{2}\right) = \frac{I_2 - I_1}{4}$$

- The charge deposited into C during A_1

$$q_c = \langle I_c \rangle t = \left(\frac{I_2 - I_1}{4}\right)\frac{T_S}{2}$$

- Capacitor equation

$$q = CV \Rightarrow V = \frac{q}{C}$$

$$\Rightarrow \Delta V = \frac{\Delta q}{C} = \left(\frac{I_2 - I_1}{4}\right)\frac{T_S}{2C}$$

so

$$V_{RC} = \Delta V_C = \frac{(I_2 - I_1) T_S}{8C}$$

Total Ripple is

$$V_R = V_{RC} + V_{RR}$$

Summary - Buck Regulator in continuous mode

$$V_O = \frac{t_{on}}{T} V_D = D V_D$$

$$I_2 - I_1 = \frac{V_d - V_o}{L} t_{on}$$

$$\frac{I_1 + I_2}{2} = I_o$$

$$I_o > \left(\frac{V_D - V_O}{2L} \right) t_{on}$$

$$V_{RR} = (I_2 - I_1) R_S$$

$$V_{RC} = \frac{(I_2 - I_1) T_S}{8C}$$

$R_S C = 50$ to $80 \mu s$

EE 456

Buck Regulator Design - Continuous Mode Operation

Define useful units for Electrical Engineering

$\mu s \equiv 10^{-6} \cdot sec$

Specify Input Voltage $V_D := 15 \cdot volt$

Specify Output Voltage $V_o := 5 \cdot volt$

Specify Switching Frequency $F_S := 20 \cdot kHz$

$T_S := \frac{1}{F_S}$ $\quad T_S = 50 \circ \mu s$

Find on time assuming continuous mode operation. $t_{on} := \frac{V_o}{V_D} \cdot T_S$ $\quad t_{on} = 16.667 \circ \mu s$

Specify the Max output Current $I_o := 1 \cdot amp$

Design the buck regulator so that it operates in the continuous mode for currents down to 10% of the max current.

$I_{o\_min} := 0.1 \cdot I_o$ $\quad I_{o\_min} = 0.1 \circ amp$

Find L so that the buck regulator operates in continuous mode for the min output current

$$L := \frac{V_D - V_o}{2 \cdot I_{o\_min}} \cdot t_{on} \qquad L = 833.333 \circ \mu H$$

Choose the next size std. inductor $L := 1000 \cdot \mu H$

With chosen inductor, find min current for continuous operation

$$I_{o\_min} := \frac{V_D - V_o}{2 \cdot L} \cdot t_{on} \qquad I_{o\_min} = 83.333 \circ mA$$

For the Max output current, find I1 and I2

$I_1 := I_o$ $\qquad$ $I_2 := I_o$

Given

$$\frac{I_2 + I_1}{2} = I_o$$

$$I_2 - I_1 = \frac{V_D - V_o}{L} \cdot t_{on}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} := \text{find}(I_1, I_2)$$

$I_2 = 1.083 \cdot \text{amp}$ $\qquad$ $I_1 = 0.917 \cdot \text{amp}$

$I_2 - I_1 = 166.667 \cdot \text{mA}$

For the Min output current, find I1 and I2

$I_{1\_} := I_{o\_min}$ $\qquad$ $I_{2\_} := I_{o\_min}$

Given

$$\frac{I_{2\_} + I_{1\_}}{2} = I_{o\_min}$$

$$I_{2\_} - I_{1\_} = \frac{V_D - V_o}{L} \cdot t_{on}$$

$$\begin{bmatrix} I_{1\_} \\ I_{2\_} \end{bmatrix} := \text{find}(I_{1\_}, I_{2\_})$$

$I_{2\_} = 0.167 \cdot \text{amp}$ $\qquad$ $I_{1\_} = 0 \cdot \text{amp}$

$I_{2\_} - I_{1\_} = 166.667 \cdot \text{mA}$

Choose the filter capacitor.

Assume that the major component of the ripple comes from the capacitor ESR

Specify the ripple due to the ESR $\quad V_{RR} := 10 \cdot \text{mV}$

$$ESR := \frac{V_{RR}}{(I_2 - I_1)} \qquad ESR = 0.06 \cdot \Omega$$

For all electrolytic caps, assume that ESR*C=80µs

$$C := \frac{80 \cdot \mu s}{ESR} \qquad C = 1.333 \cdot 10^3 \cdot \mu F$$

Choose the next size std capacitor $\quad C := 2200 \cdot \mu F$

Calculate the new ESR with the chosen capacitor

$$ESR := \frac{80 \cdot \mu s}{C} \qquad ESR = 0.036 \circ \Omega$$

Find the ripple due to the capacitor:

$$V_{RC} := \frac{(I_2 - I_1) \cdot T_S}{8 \cdot C} \qquad V_{RC} = 0.473 \circ mV$$

FInd the ripple due to the ESR

$$V_{RR} := (I_2 - I_1) \cdot ESR \qquad V_{RR} = 6.061 \circ mV$$

Calculate the RMS ripple current for the capacitor.

Assume a triangular waveform with a 50% duty cycle - Worst case.

$$A := \frac{I_2 - I_1}{2} \qquad A = 0.083 \circ amp$$

$$I(t) := \frac{A}{\left(\frac{T_S}{4}\right)} \cdot t$$

$$I_{rms} := \sqrt{\frac{4}{T_S} \left[\int_{0 \cdot sec}^{\frac{T_S}{4}} I(t)^2 \, dt \right]}$$

$$I_{rms} = 0.048 \circ amp$$

Summary

Inductor

$L = 1 \circ mH$ Peak Current Rating $I_2 = 1.083 \circ amp$ Avg Current Raiting $I_o = 1 \circ amp$

Capacitor

$C = 2.2 \cdot 10^3 \circ \mu F$ RMS Ripple Current Rating $I_{rms} = 0.048 \circ amp$

Regulator Specs

Maximim Current $I_o = 1 \circ amp$ Period $T_S = 50 \circ \mu s$ Switching Frequency $F_S = 20 \circ kHz$

Switch on time $t_{on} = 16.667 \circ \mu s$ Output Voltage $V_o = 5 \circ volt$ Input Voltage $V_D = 15 \circ volt$

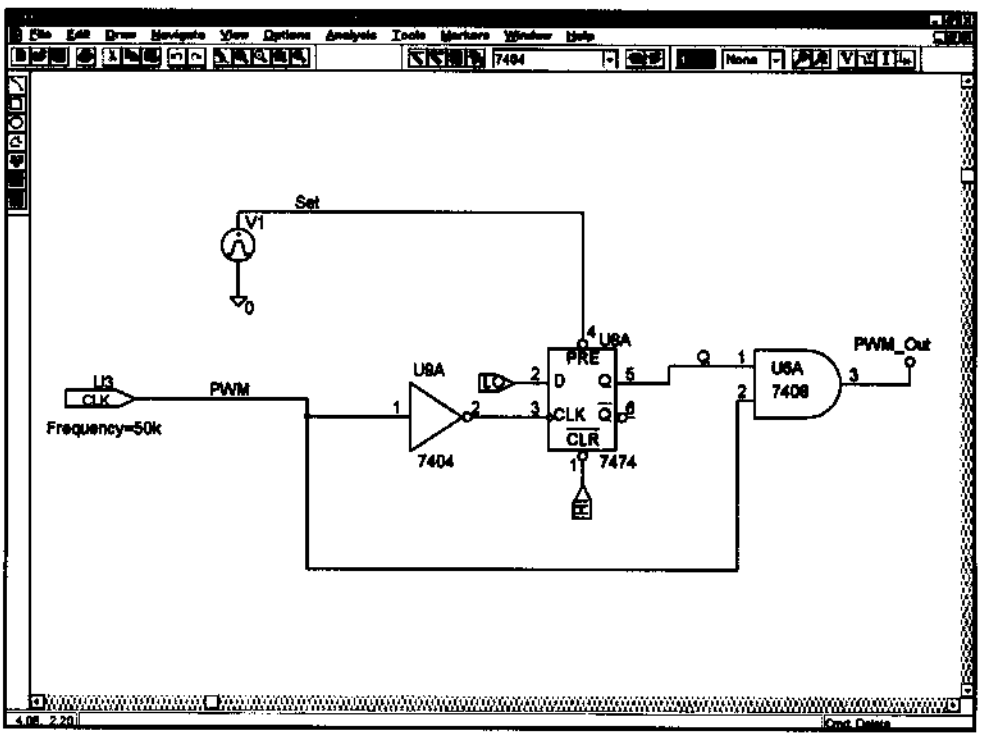
Currents $I_2 = 1.083 \circ amp$ $I_1 = 0.917 \circ amp$ $I_2 - I_1 = 166.667 \circ mA$

(111)

Add Double Pulse Suppression

- Set a flip-flop at the beginning of a PWM cycle.
- Negative edge from comparator clears the flip-flop.
- Attempt to use as few components as possible.

Advanced PSpice Workshop – June 7-9, 2000



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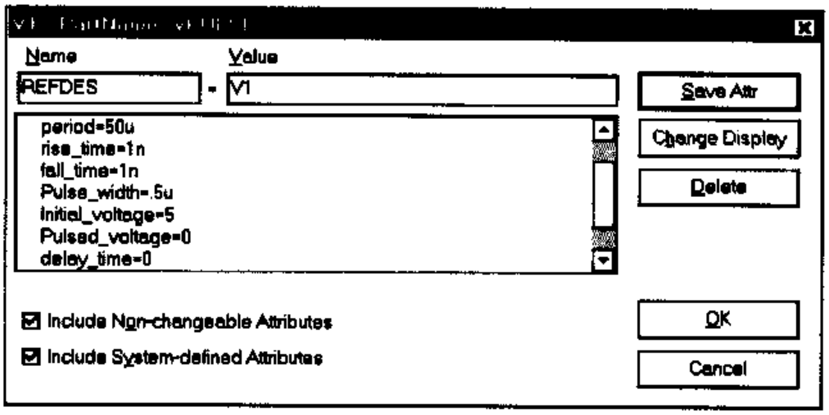
Attributes Of The Set Signal (V1)



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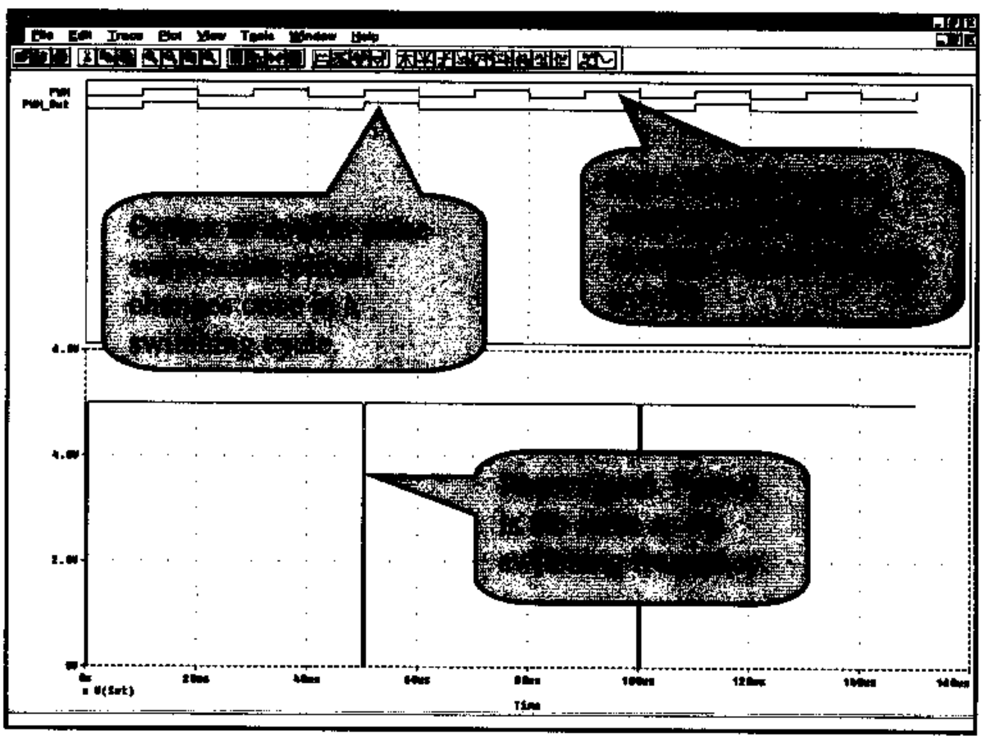

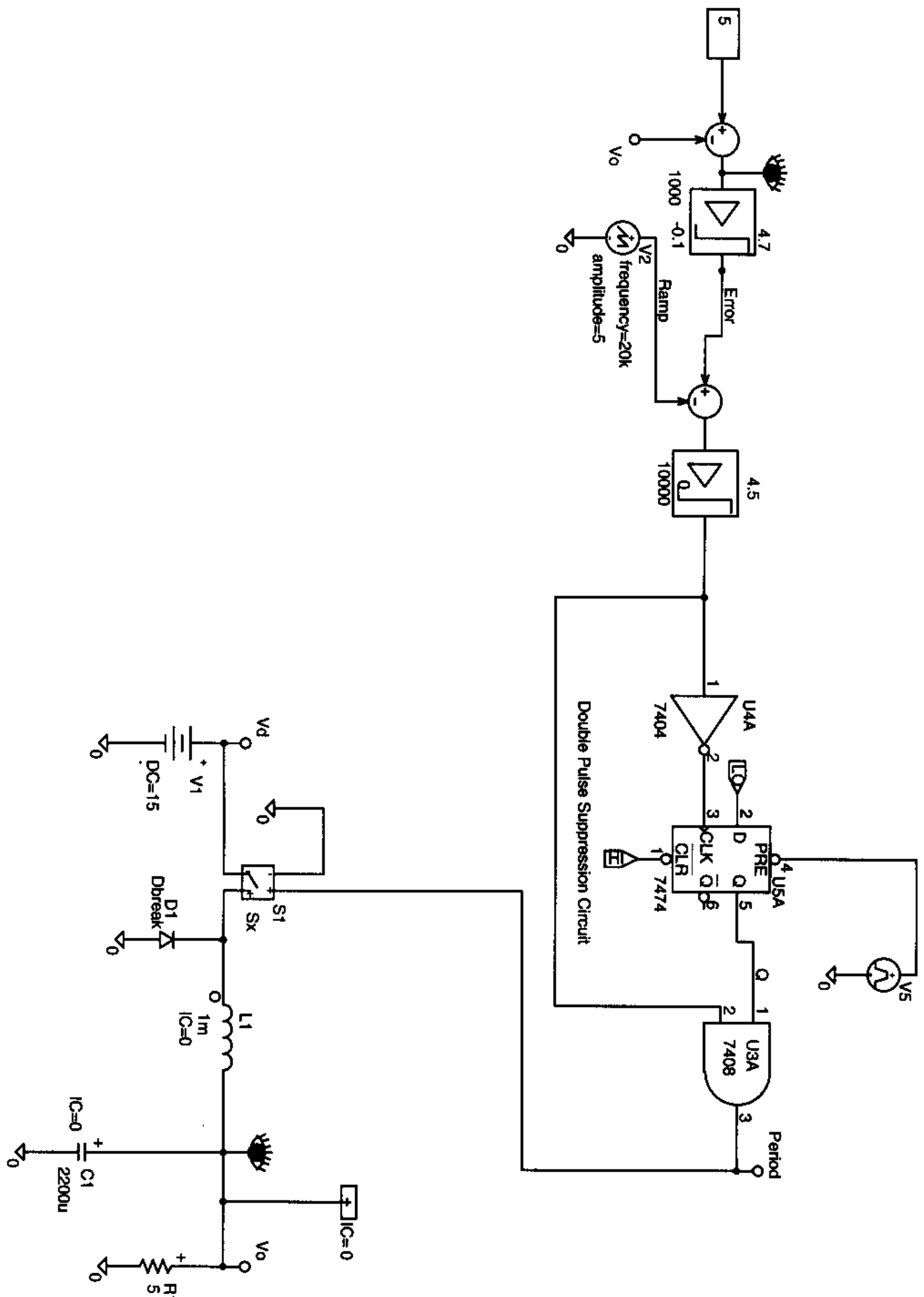
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Vo
1000 -0.1
4.7
Error
V2
frequency=20k
amplitude=5
Ramp
4.5
10000
U4A
7404
LO
HI
PRE
D
CLK
CLR
Q
U5A
7474
V5
Q
U3A
7408
Period
Double Pulse Suppression Circuit
Vd
V1
DC=15
S1
Sx
D1
Dbreak
L1
1m
IC=0
C1
2200u
IC=0
IC=0
Vo
R1
5

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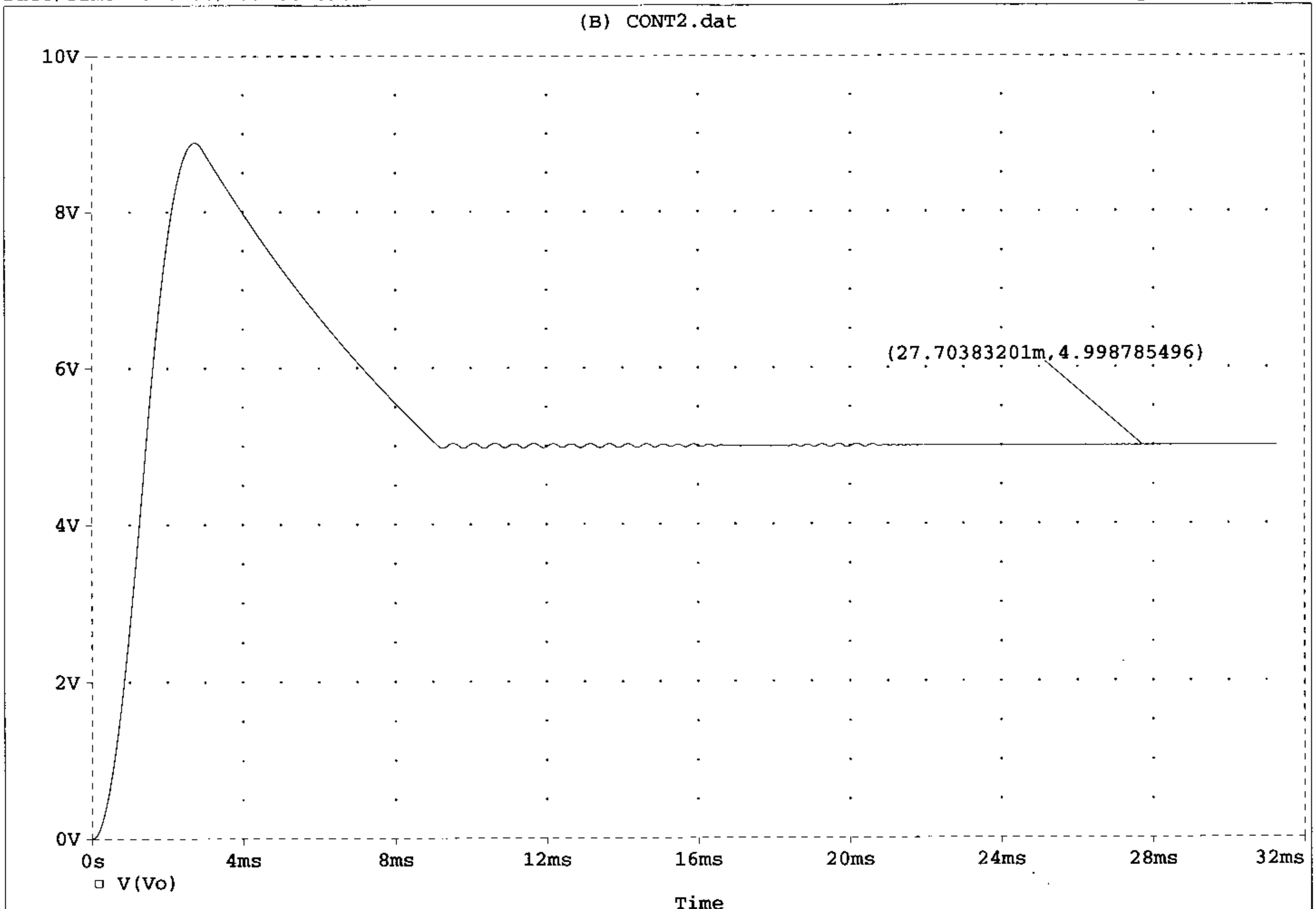
Date/Time run: 06/19/100 09:49:15 Temperature: 27.0



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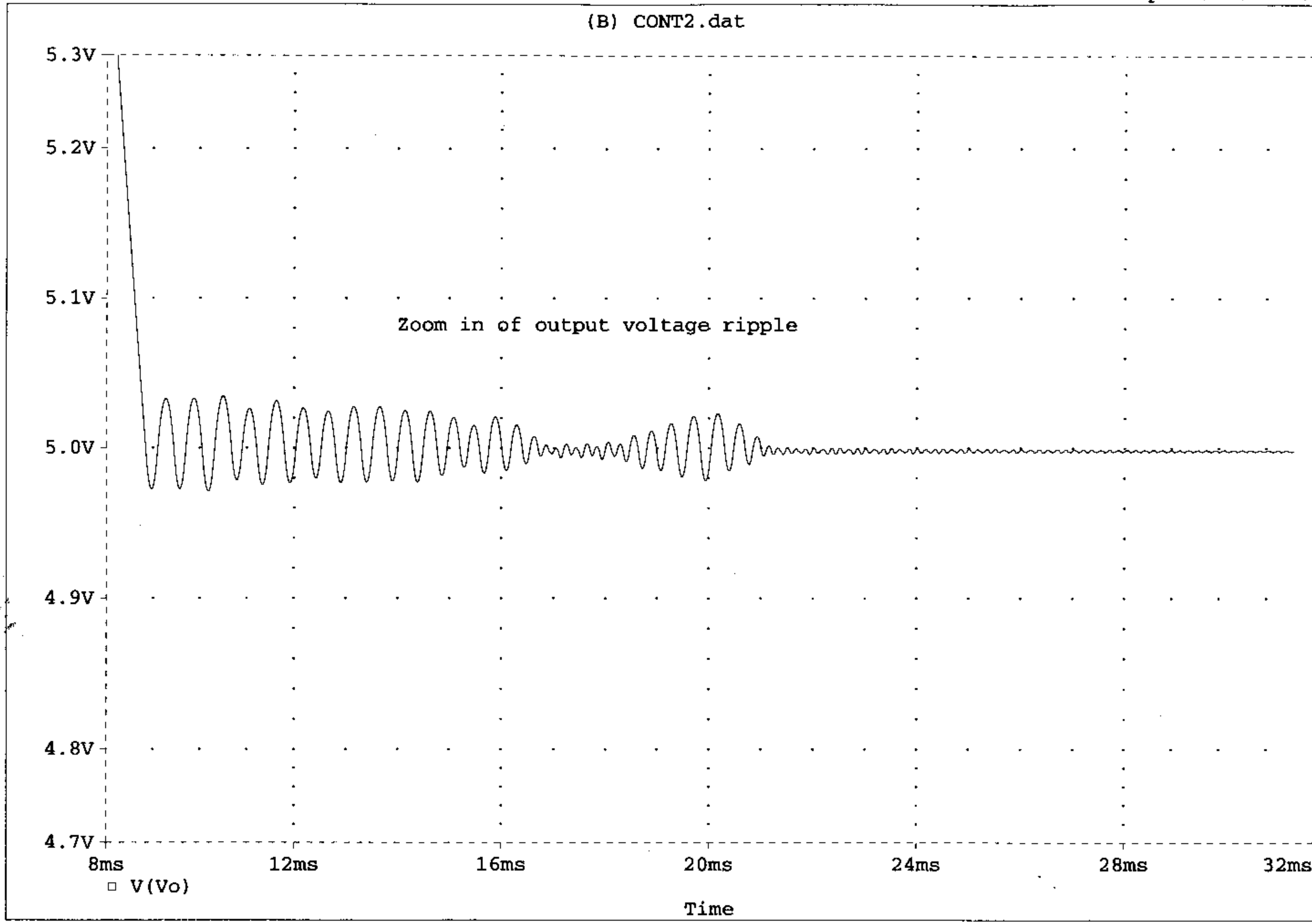

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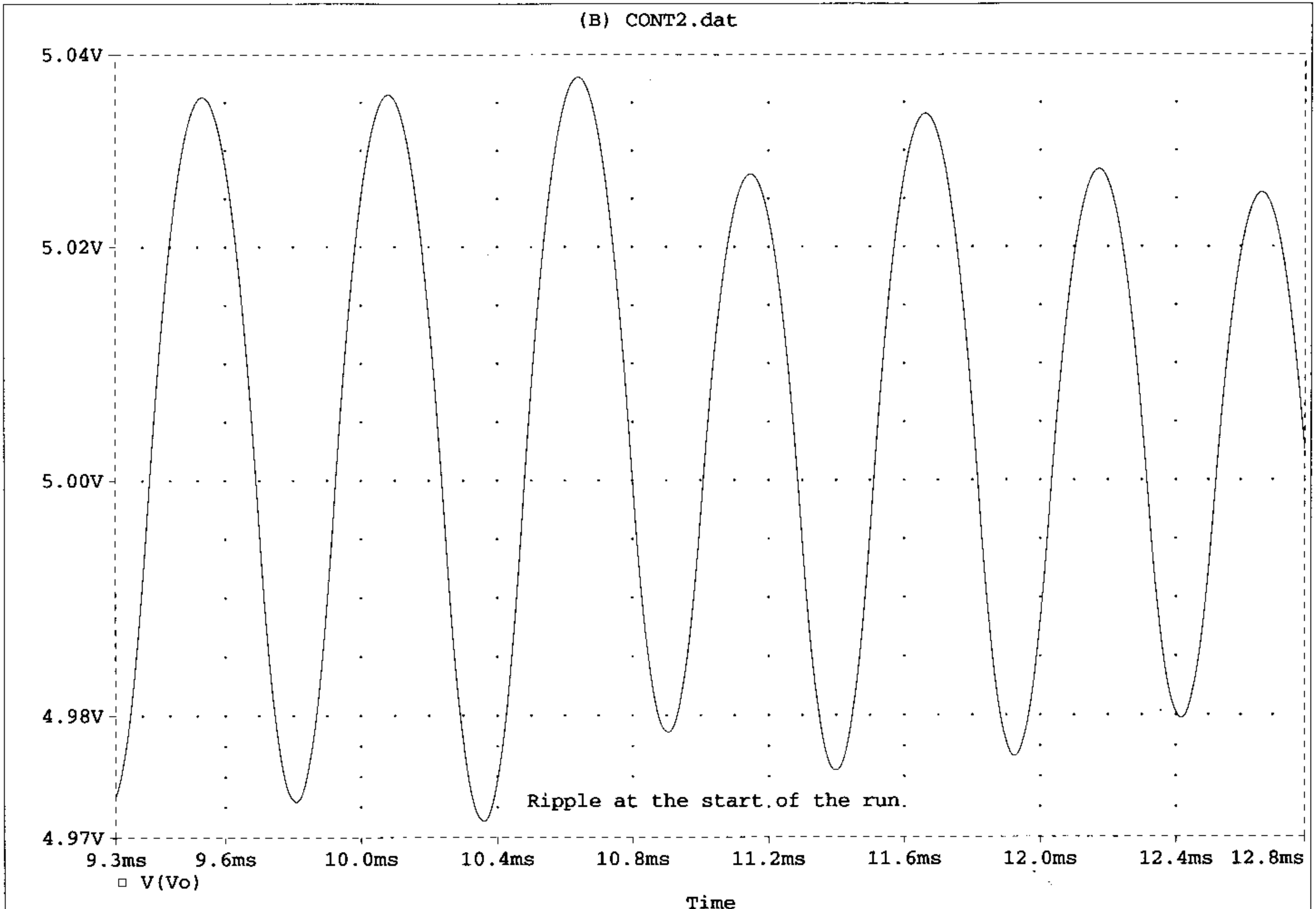

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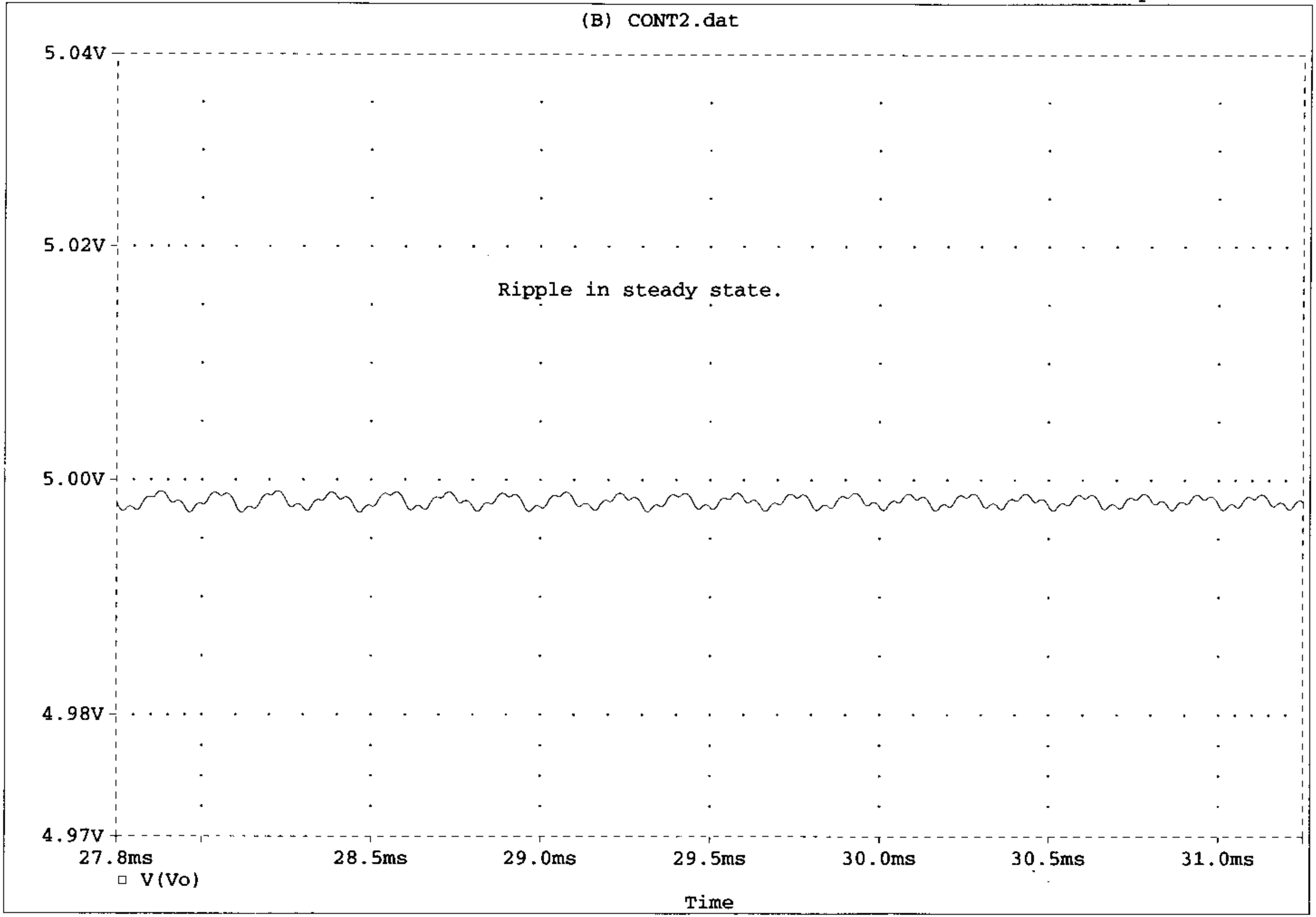

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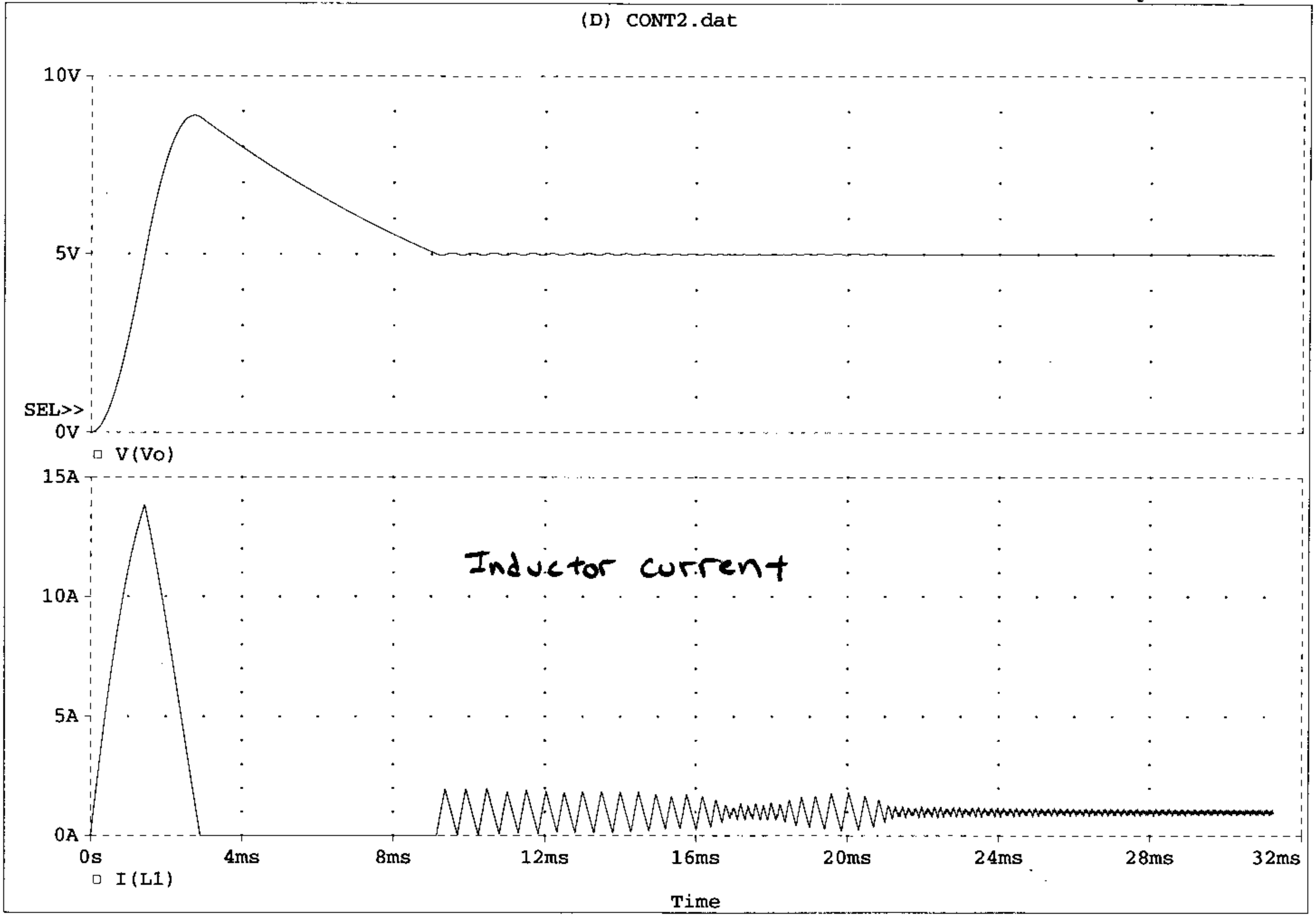

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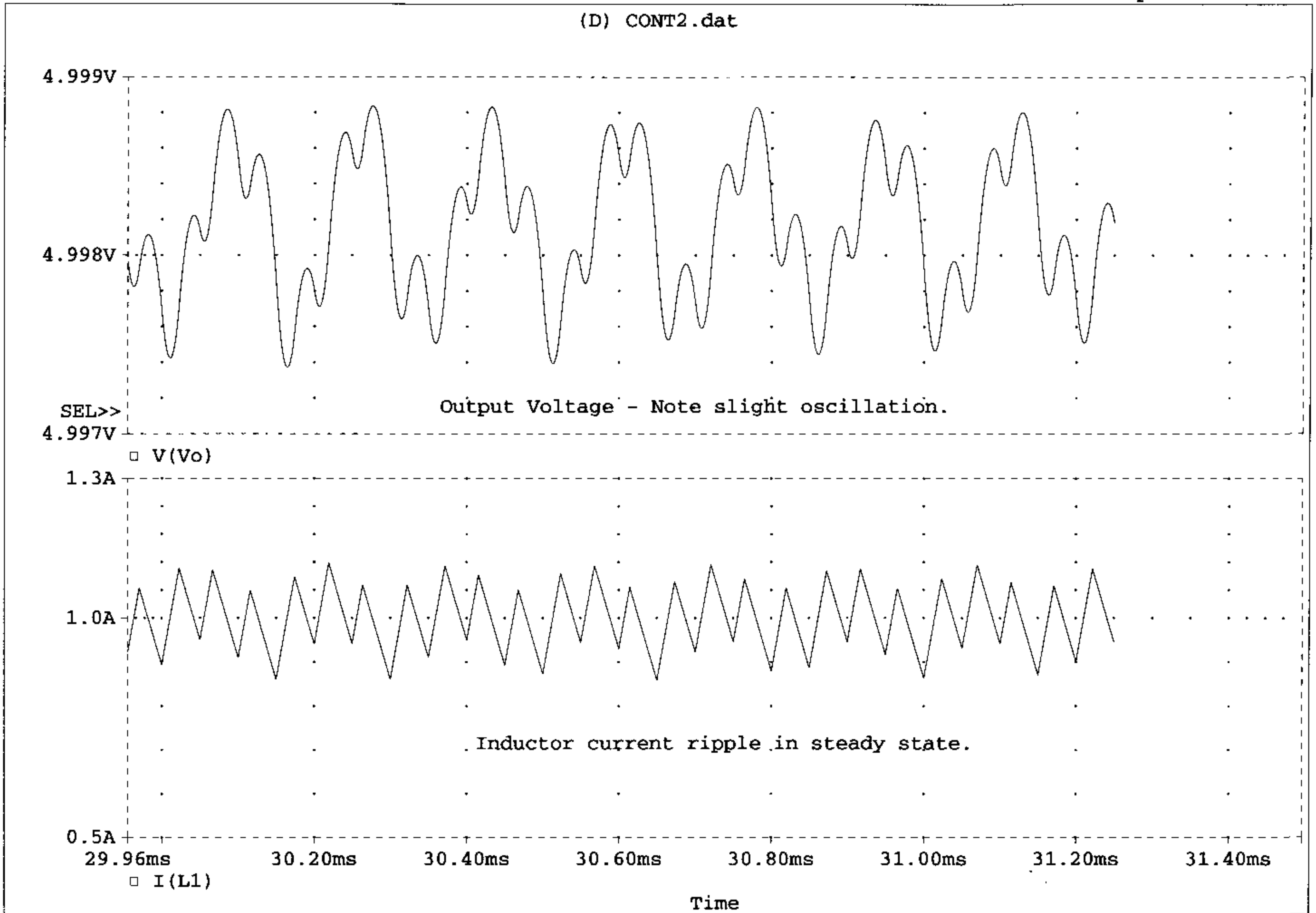

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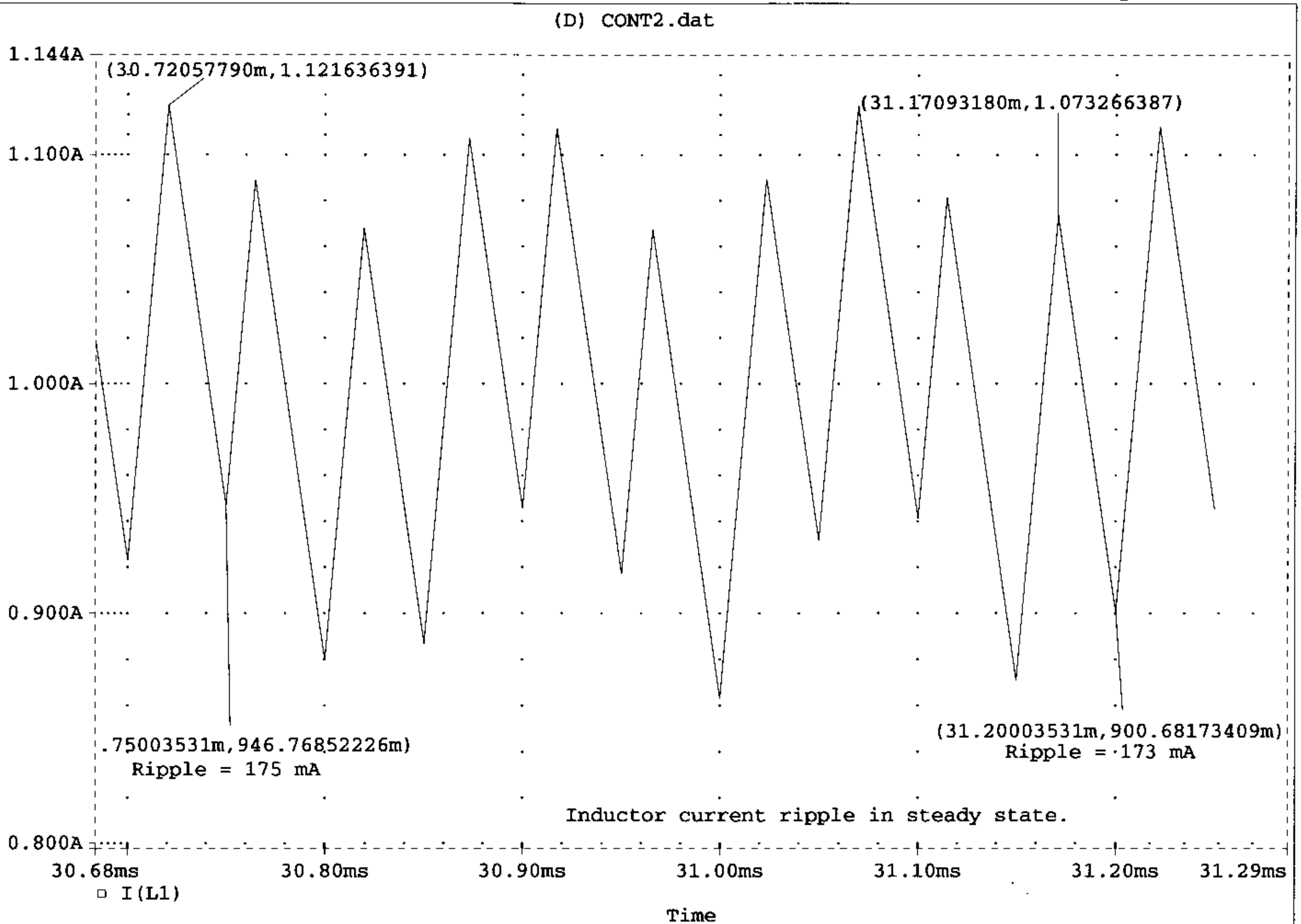

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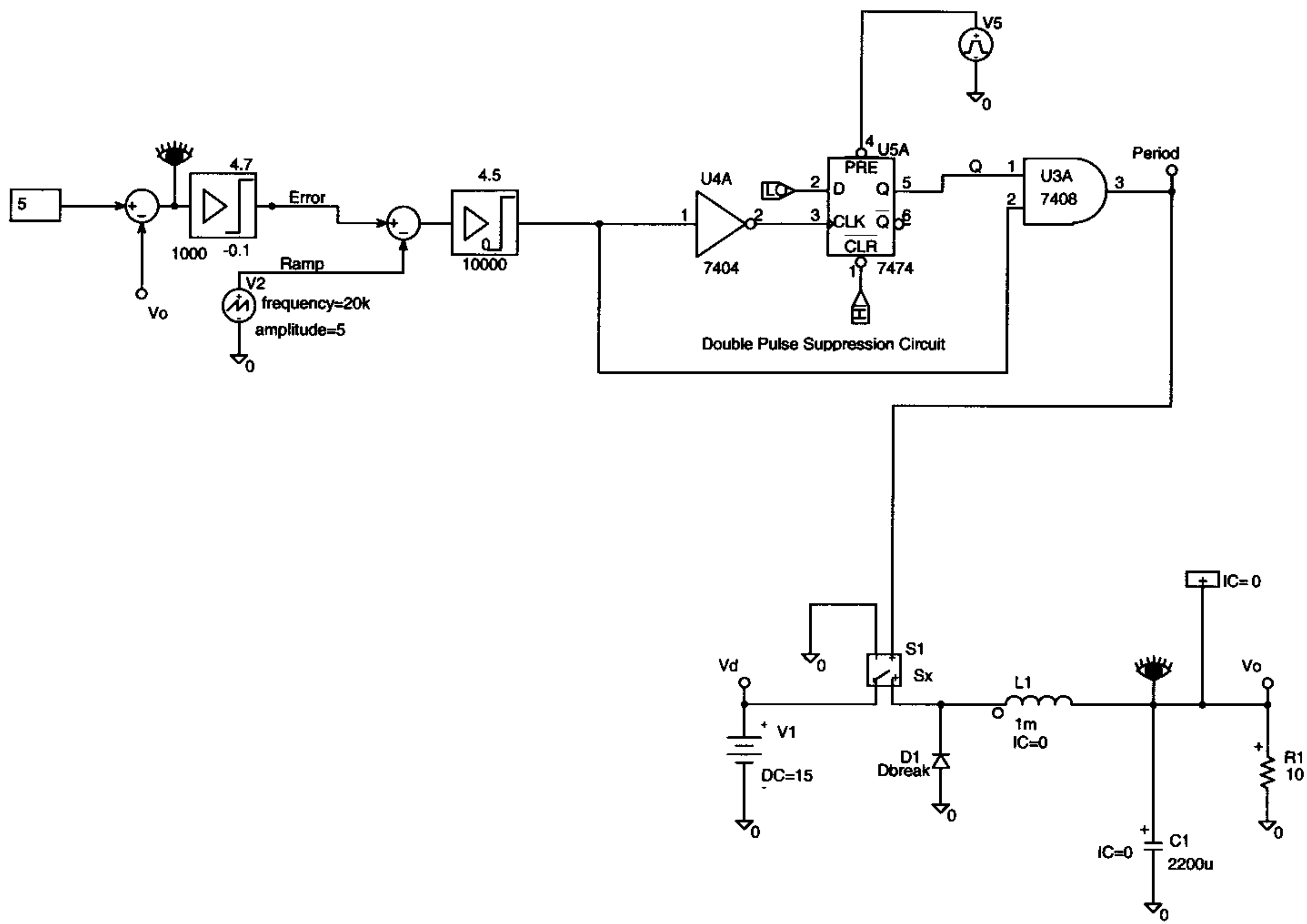

5
Vo
1000
-0.1
4.7
Error
Ramp
V2
frequency=20k
amplitude=5
0
4.5
10000
0
U4A
1
2
7404
LO
2
3
D
CLK
PRE
CLR
Q
Q
4
U5A
5
6
1
7474
HI
V5
0
Double Pulse Suppression Circuit
Q
1
2
U3A
7408
3
Period
Vd
V1
DC=15
0
S1
Sx
0
D1
Dbreak
0
L1
1m
IC=0
IC=0
C1
2200u
0
IC= 0
Vo
R1
10
0

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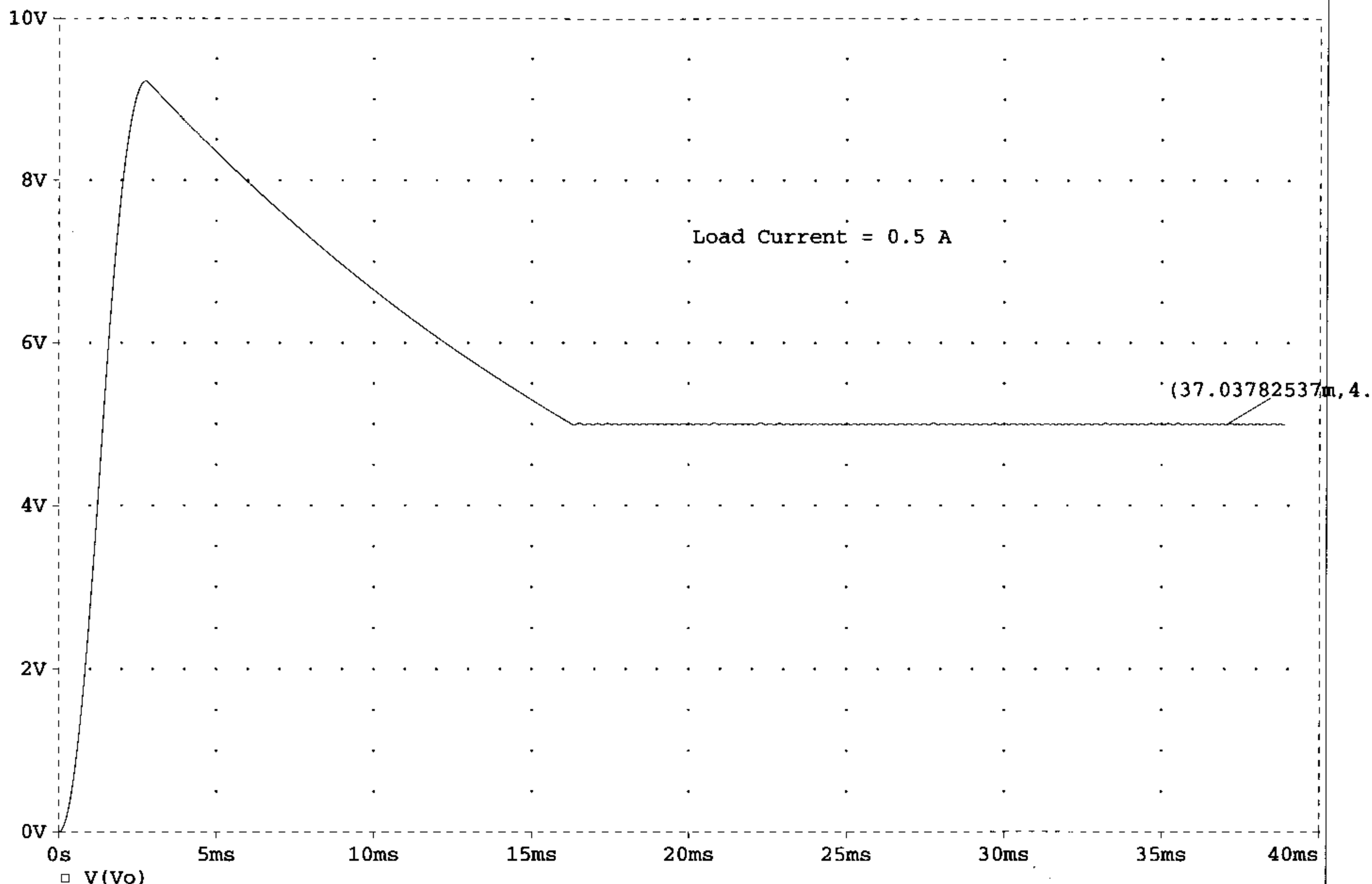
Date/Time run: 06/19/100 10:07:36 Temperature: 27.0

(C) CONT2.dat



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94A

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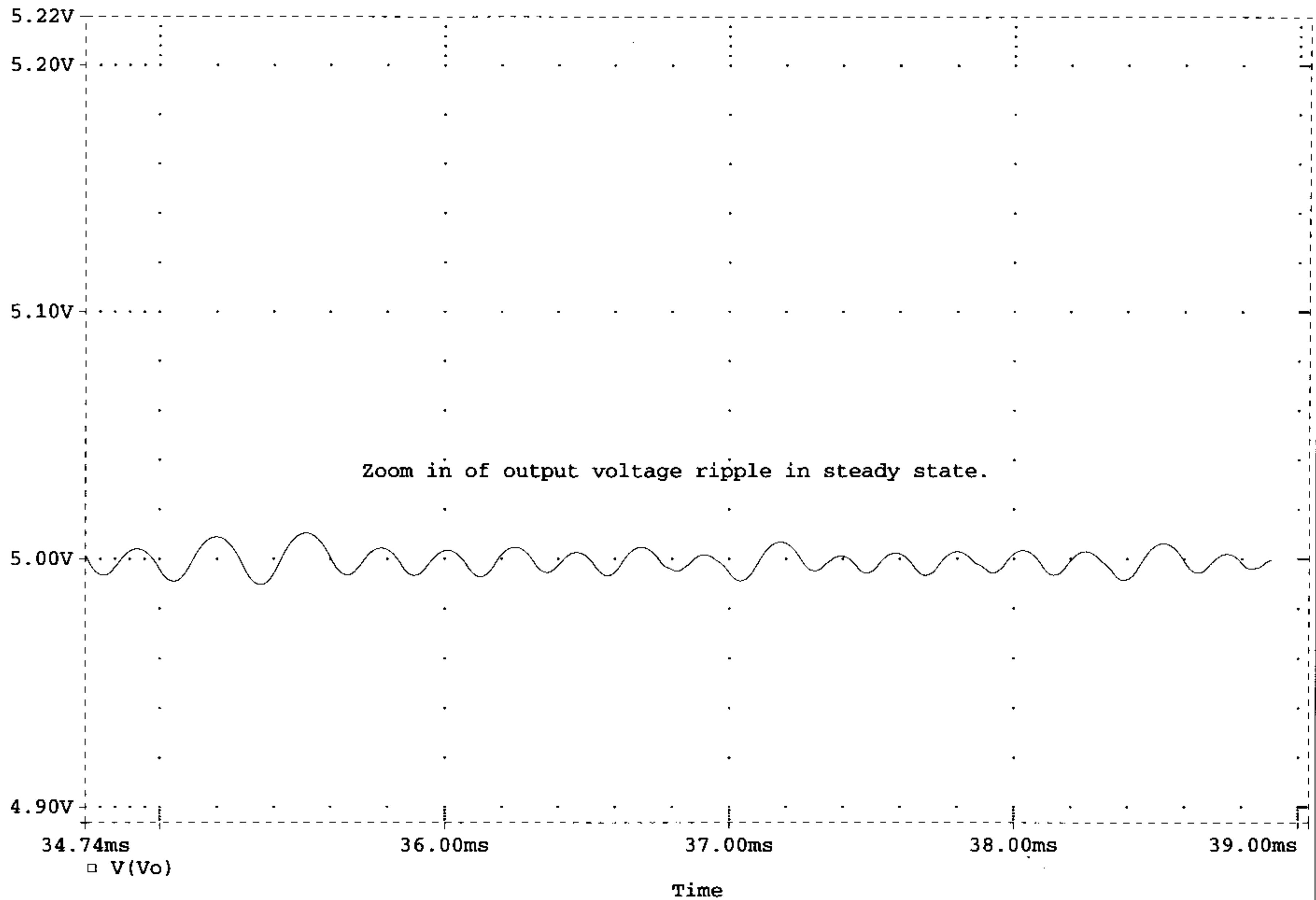

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94B

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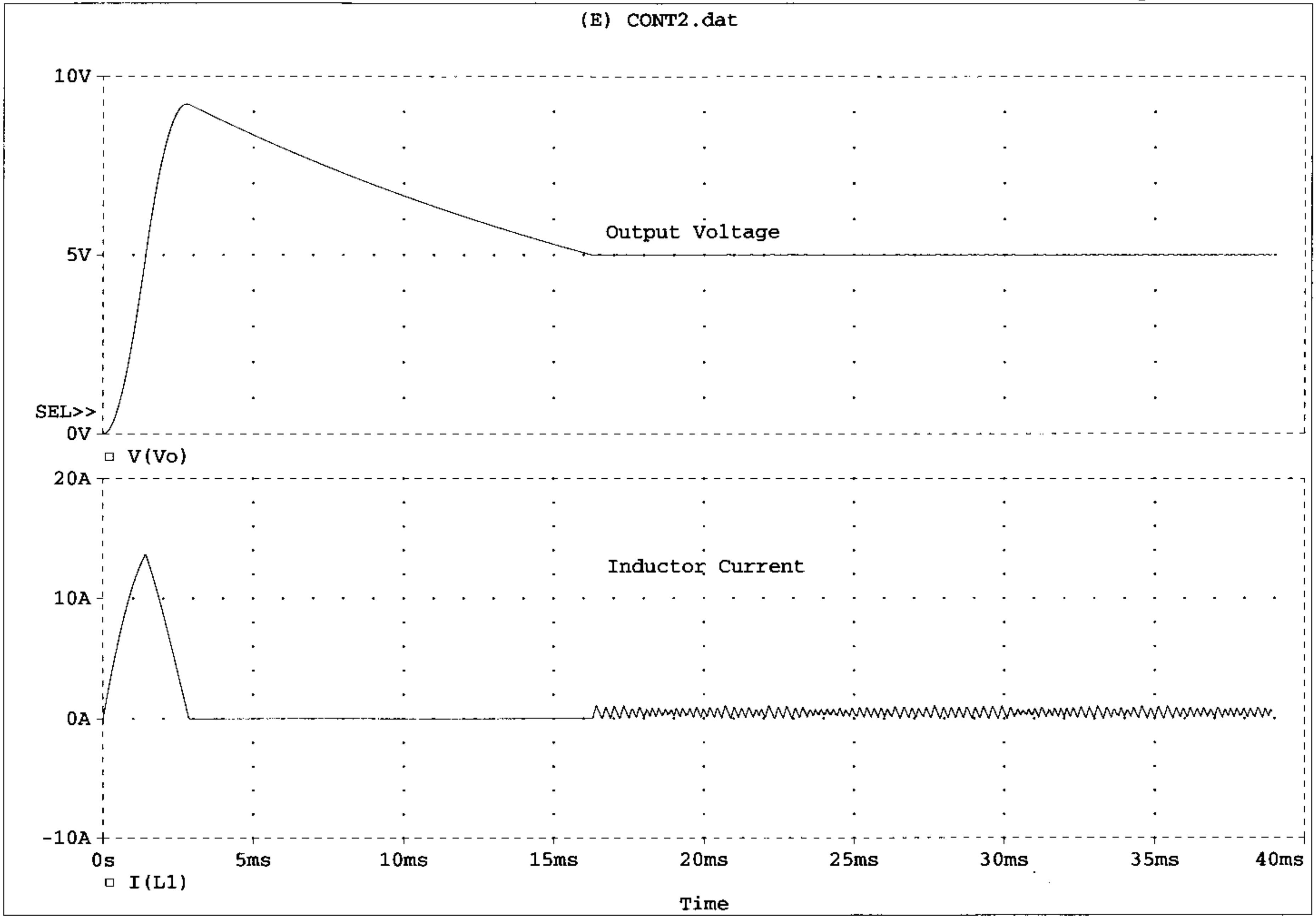

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94C

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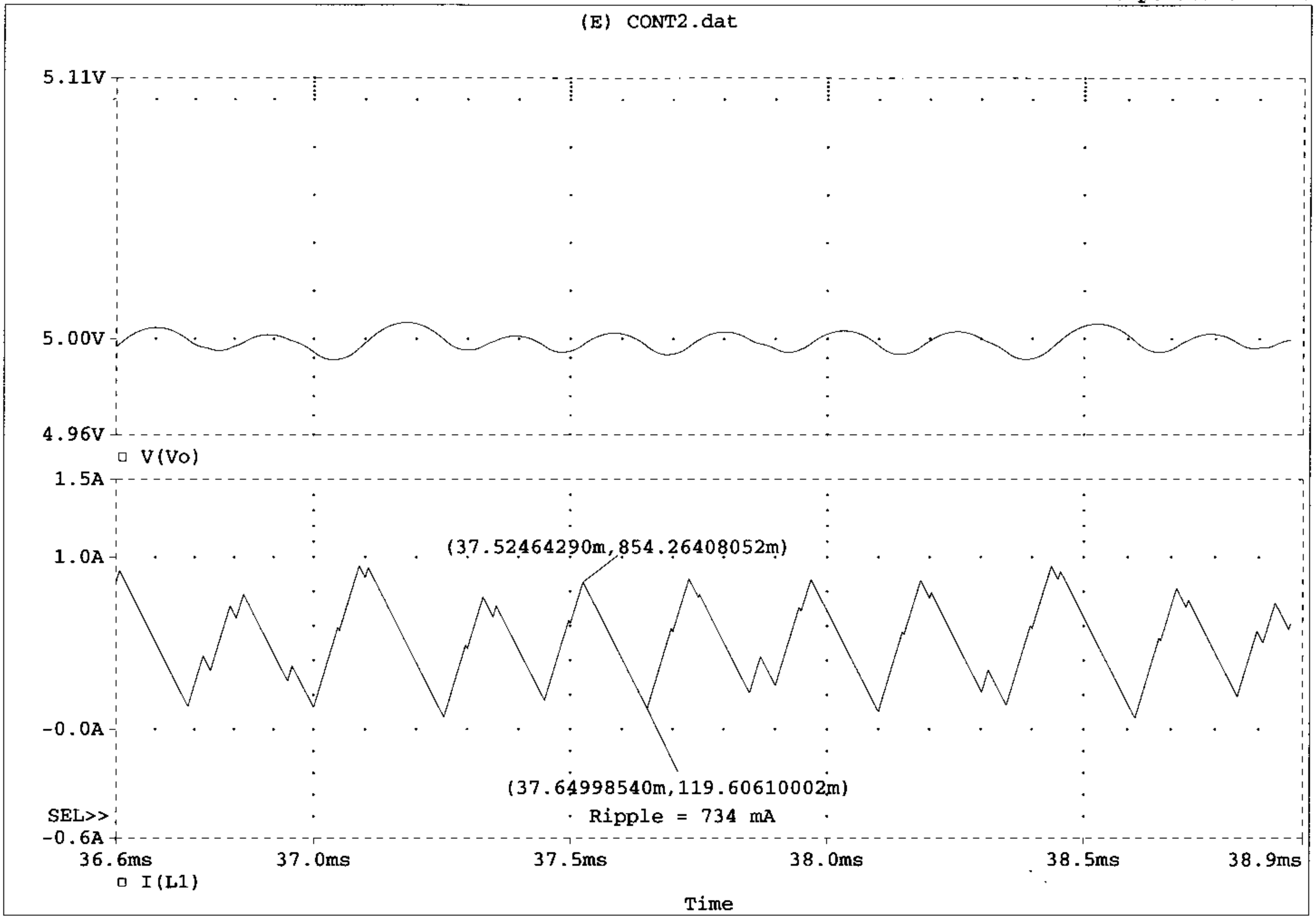

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94D

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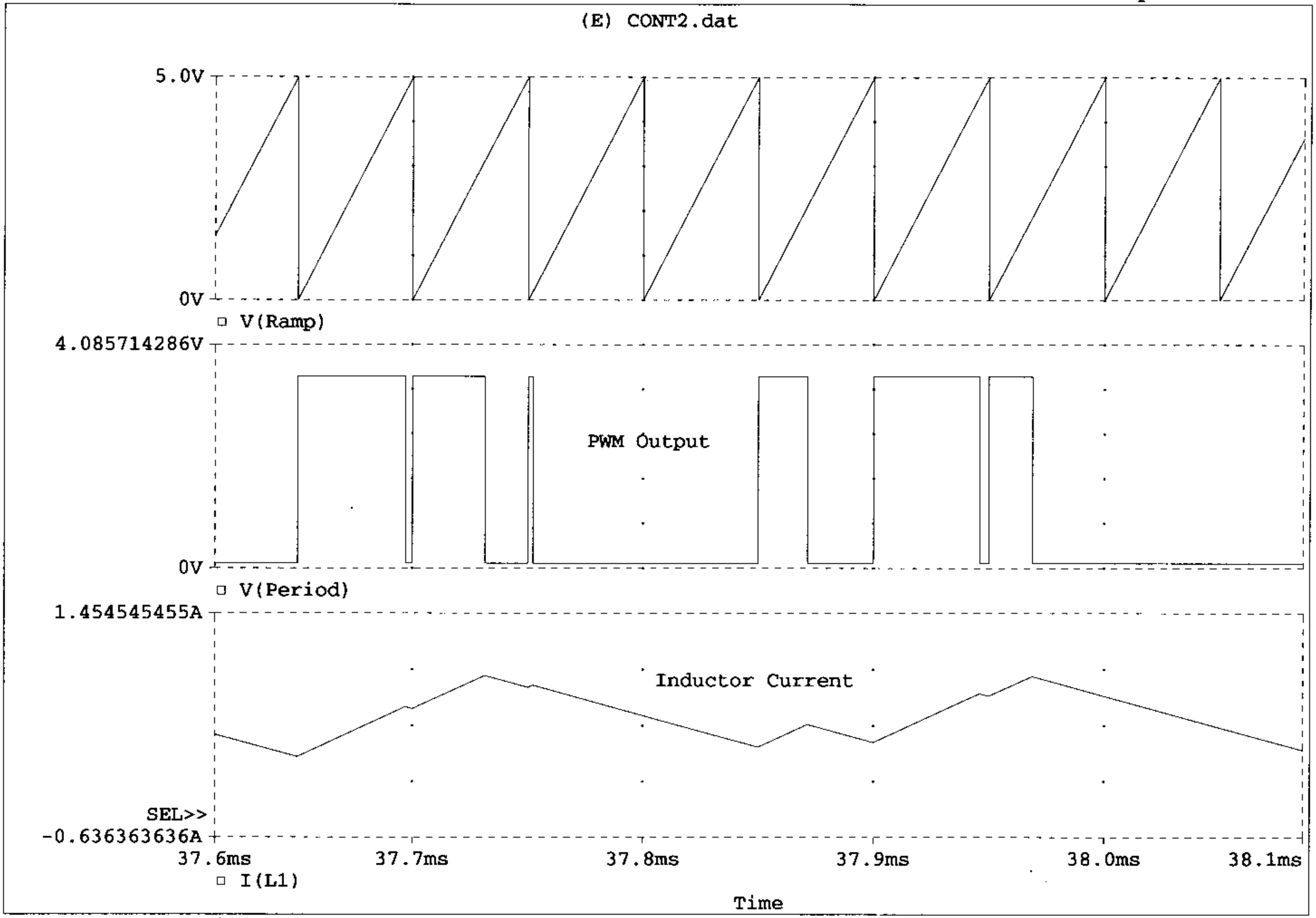

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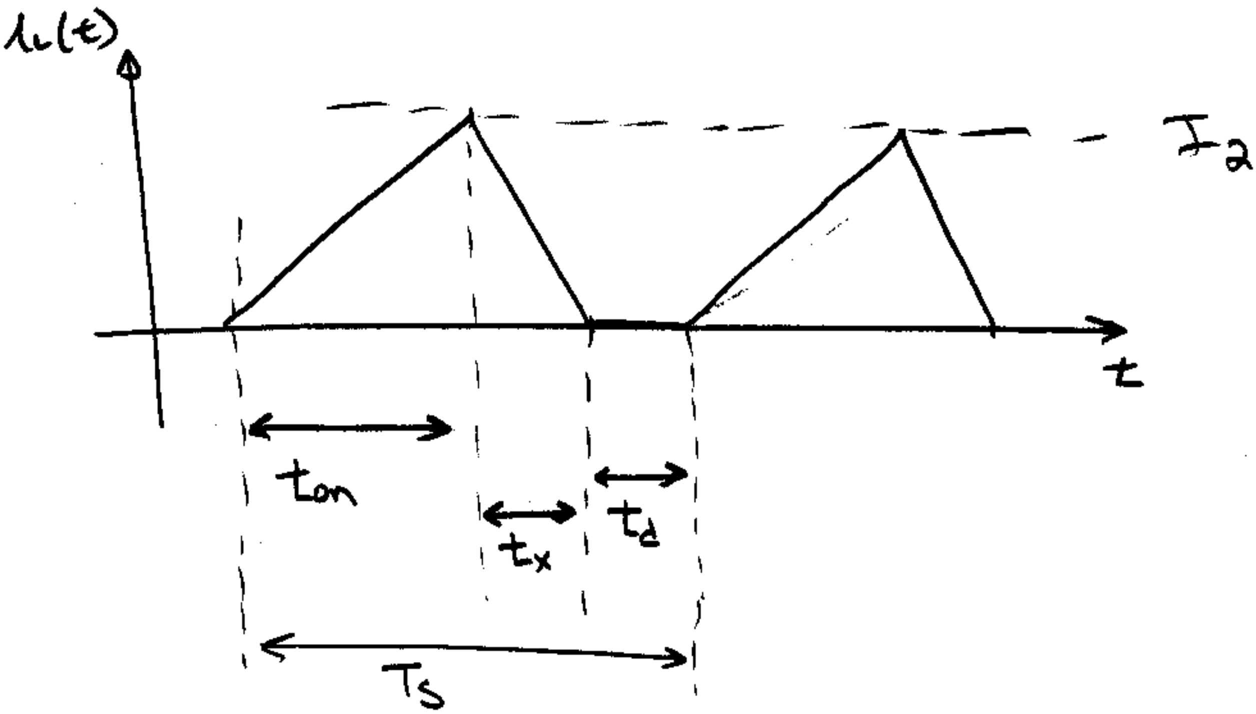
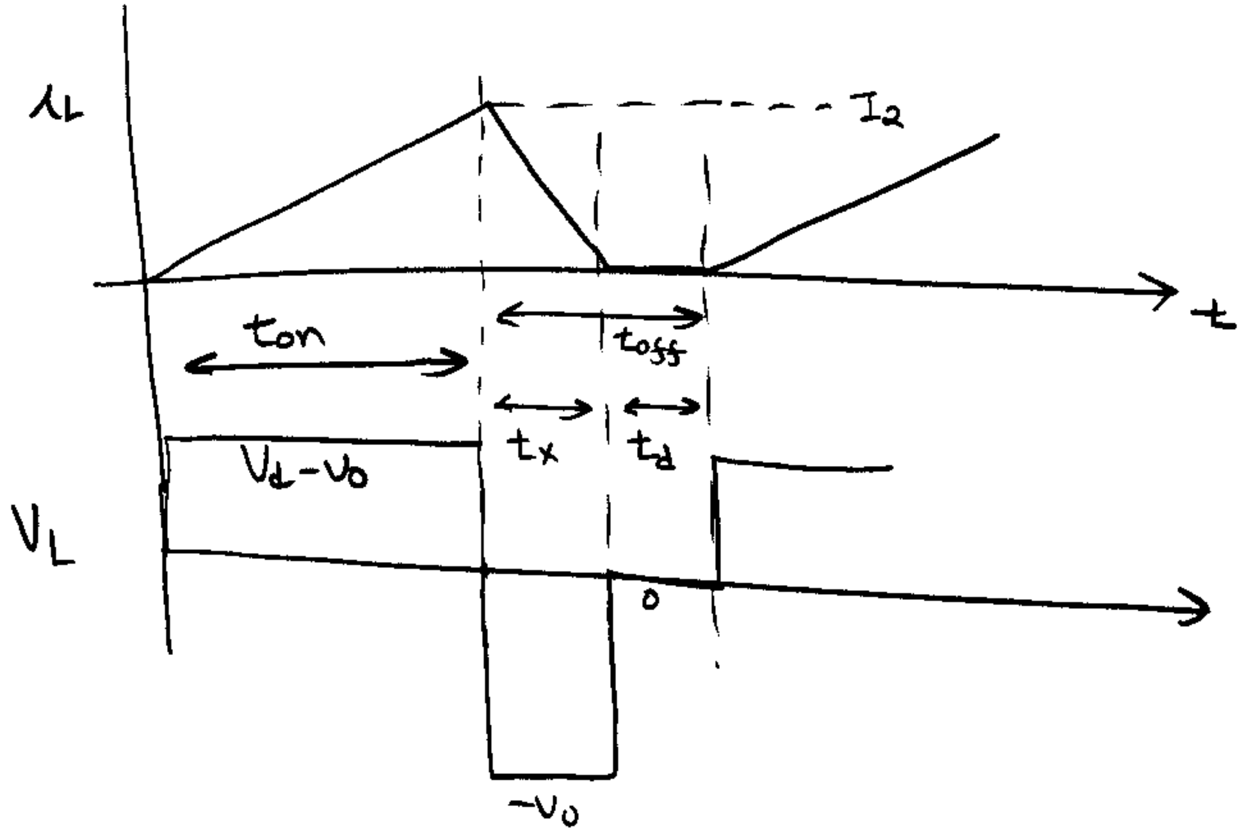
Buck Regulator - Discontinuous mode

as $I_o \downarrow$, $I_1 \to 0$

Inductor current is zero sometimes



In discontinuous mode, All energy stored in the inductor is dumped to output



for Average inductor current to be const,

$$\int_{T_s} V_L(t)\,dt = 0 \quad \text{for one period}$$

so

$$(V_d - V_o)\,t_{on} + (-V_o)\,t_x + 0\,t_d = 0$$

$$V_o = V_d\left[\frac{t_{on}}{t_{on}+t_x}\right] \qquad (1)$$

For an inductor

$$V_L = L \frac{di_L(t)}{dt} \Rightarrow i_L(t) = \frac{1}{L}\int V_L \, dt$$

For ton

$$I_2 = \frac{1}{L}(V_d - V_o)\, t_{on} \quad (2)$$

For tx

$$I_2 = \frac{1}{L}(V_o)\, t_x \quad (3)$$

Find average current

During ton and tx

$$\langle \hat{i}_L \rangle = I_2/2 \quad \text{- avg inductor current}$$

During Ts

$$\langle i_L \rangle = \langle \hat{i}_L \rangle \left(\frac{t_{on} + t_x}{T_s} \right)$$

so

$$\langle i_L \rangle = \frac{I_2}{2}\left(\frac{t_{on} + t_x}{T_s} \right) = I_o \quad (4)$$

Solve equation (4) for $t_{on} + t_x$

$$t_{on} + t_x = \frac{2 I_o T_s}{I_2}$$

sub into (1)

$$V_o = V_d \left[\frac{t_{on}}{\frac{2 I_o T_s}{I_2}} \right]$$

OR

$$V_o = V_o \left[\frac{t_{on} I_2}{2 I_o T_s} \right] \quad (5)$$

Now eliminate I_2 with eq. (2)

$$V_o = V_d \left[\frac{t_{on} (V_d - V_o) t_{on}}{2 I_o T_s L} \right]$$

So

$$V_o\left[1+\frac{V_D\,t_{on}^2}{2I_oT_sL}\right]=\frac{t_{on}^2\,V_D^2}{2I_oT_sL}$$

$$V_o=\frac{t_{on}^2\,V_D^2}{2I_oT_sL+V_D\,t_{on}^2}$$

OR

$$\frac{V_o}{V_D}=\frac{D^2V_D}{\frac{2I_oL}{T_s}+D^2V_D}$$

$$I_o\le\left(\frac{V_D-V_o}{2L}\right)t_{on}$$

in Discontinuous mode – To keep V_o const with changing I_o, can change D.

$$D=\frac{t_{on}}{T_s}$$